Parallel Space-Time Finite Element Solvers

Ulrich Langer[a], **and Andreas Schafelner**[b]

[a] *Institute of Computational Mathematics, Johannes Kepler University, Altenberger Str. 69, 4040 Linz, Austria*
[b] *DK Computational Mathematics, Johannes Kepler University, Altenberger Str. 69, 4040 Linz, Austria*

Following [1], we propose locally stabilized, conforming finite element schemes on completely unstructured simplicial space-time meshes for the numerical solution of parabolic initial-boundary value problems

$$\partial_t u - \mathrm{div}_x(\nu\, \nabla_x u) = f \text{ in } Q = \Omega \times (0,T),\ u = 0 \text{ on } \Sigma = \partial\Omega \times (0,T),\ \text{and } u = u_0 \text{ on } \Sigma_0 = \Omega \times \{0\},$$

with variable coefficients $\nu(x,t)$ that may be discontinuous in space and time. Discontinuous coefficients, non-smooth boundaries, changing boundary conditions, non-smooth or incompatible initial conditions, and non-smooth right-hand sides can lead to non-smooth solutions. We present new a priori discretization error estimates for low-regularity solutions. In order to avoid reduced convergence rates appearing in the case of uniform mesh refinement, we also consider adaptive refinement procedures based on residual a posteriori error indicators. The huge system of space-time finite element equations is then solved by means of GMRES preconditioned by algebraic multigrid. In particular, in the 4d space-time case that is 3d in space, simultaneous space-time parallelization can considerably reduce the computational time. Figure 1 shows the decomposition of the space-time cylinder $Q = (0,1)^{d+1} = (0,1)^d \times (0,1)$ into 64 subdomains for parallel computing in the case $d = 2$. Figure 2 presents the convergence history of the discretization error in the energy norm for $d = 2, 3$ and different polynomial degrees p. In the case $d = 3$, the strong scaling of the solver is illustrated in Figure 3 for $N_h = 4\,601\,025$, $4\,601\,025$, and $5\,764\,801$ corresponding to $p = 1$, 2, and 3, respectively, where N_h denotes the total number of space-time unknowns. The numerical experiments were performed on the distributed memory machine RADON1 using the software library MFEM[1].

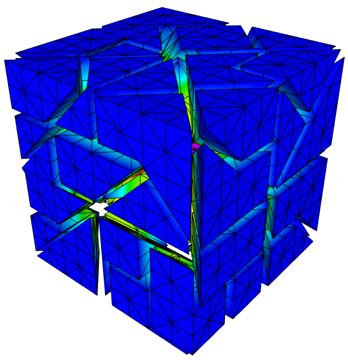

Fig. 1: Decomposition of Q.

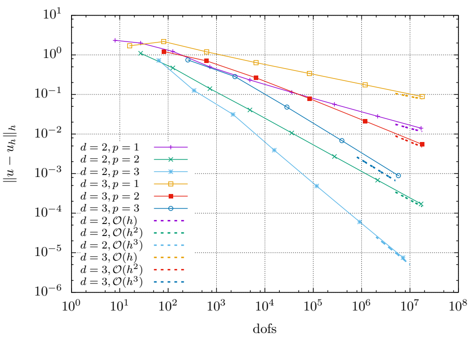

Fig. 2: Convergence rates.

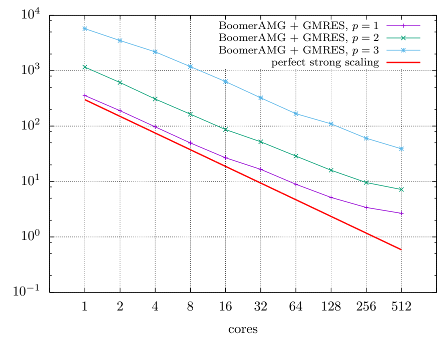

Fig. 3: Strong scaling results

We refer the reader to [1] for a comprehensive overview of relevant references to space-time methods. Furthermore, the authors would like to thank the Austrian Science Fund (FWF) for the financial support under the grant DK W1214-04.

References

[1] Langer U., Neumüller M., and Schafelner A, in: Advanced Finite Element Methods with Applications - Proceedings of the 30th Chemnitz FEM Symposium 2017, edited by T. Apel and U. Langer and A. Meyer and O. Steinbach, LNCSE, Springer, Heidelberg, 2019, to appear.

[1] http://mfem.org/